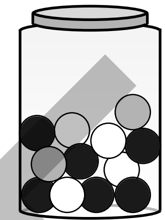
Standard 7.1A; 7.1D; 7.1E; 7.1G; 7.6A (L–M)

Probability: Representing Sample Spaces

If you roll a standard die, what are the chances of it landing on 1? If you flip a coin, what are the chances of it landing on "heads"? When you use a phrase like "what are the chances" in math, you are talking about probability. **Probability** is the likelihood that an event will happen.

Suppose you reached into the jar shown to the right and selected one marble without looking. Since you are selecting one marble at random, there is a chance you could choose any of them. Selecting a marble from the jar in a probability experiment is called an event. An **event** is a set of one or more outcomes in an experiment.



Talk About It–1: If you select one marble at random from the jar shown above, what three events are possible?

The jar above has three colors of marbles, but there are different numbers of each color. What are your chances of selecting any one of those colors if you randomly choose one marble from the jar? To answer that question, you must know about probability. Using probability, you can compare one random outcome with all possible outcomes. The set of all possible outcomes is known as the **sample space**.

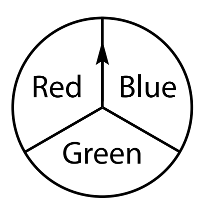
Talk About It–2: Look at the jar of marbles again. If you reach into the jar and select one marble, what is the sample space? In other words, what are all the possible outcomes?

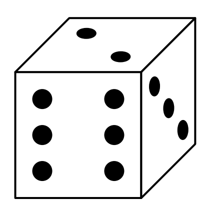
Selecting one marble from the jar is a simple event. A **simple event** has exactly one outcome. For example, when you select a marble from the jar, there can be only one outcome; you will select a gray, white, or black marble. You can write the sample space as shown below.

{gray, white, black}

Compound events, on the other hand, have more than one outcome. Consider the following example.

Julia plays a game with a spinner and a die, like the ones shown to the right. Spinning the spinner results in a single outcome. Rolling the die results in a single outcome. However, if Julia spins the spinner and rolls the die on each turn, she produces two outcomes for a single event.





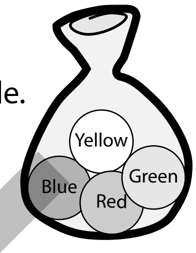
If Julia spins the spinner and rolls the die on each turn, what are all the possible outcomes? What is the sample space? To determine the events that will make up the sample space, Julia can show the data in an organized list or a tree diagram. Let's look at each one.

continue to next page

Standard 7.1A; 7.1D; 7.1G; 7.6C; 7.6D (L–M)

Understanding Compound Events II

We can build on your knowledge of compound events by looking at another example.

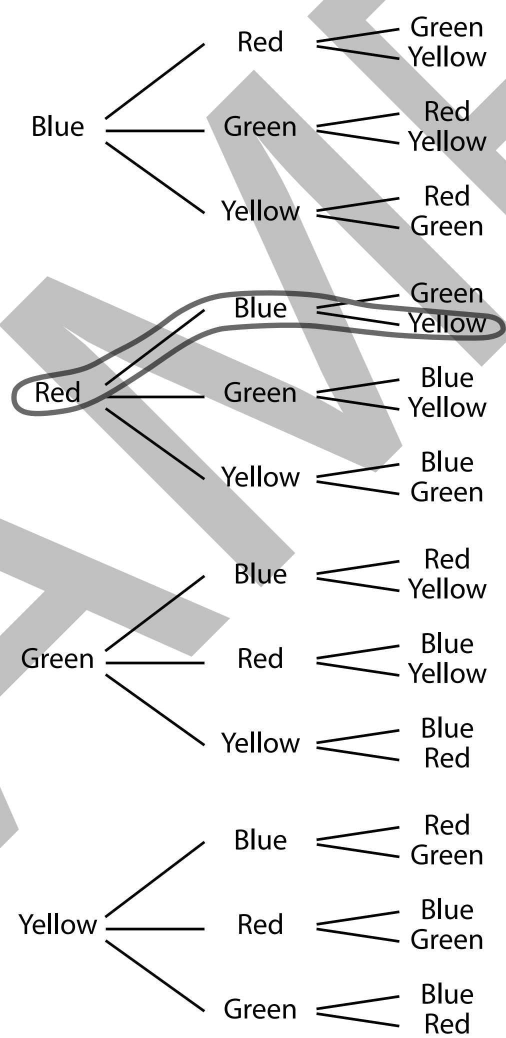


Cody has the bag of marbles shown to the right. He selects three marbles from the bag, one at a time. He does not replace the marbles after each selection.

What is the probability that Cody will select a red, a blue, and a yellow marble (in that order)?

First, determine whether the three events are independent or dependent. Ask yourself: Does the outcome of Cody's first selection affect the selections he makes after that? The answer is yes. Thus, the events are dependent.

To find the probability of Cody selecting, in order, a red, a blue, and a yellow marble, or *P(R, B, Y)*, we can use one of the three methods you learned. We will use a tree diagram to find all of the outcomes for this problem.



From the tree diagram, we can see that there is 1 outcome out of 24 that Cody will select, in order, a red, a blue, and a yellow marble from the bag. That gives us a probability of $\frac{1}{24}$.

continue to next page

Standard 7.1A; 7.1F; 7.6H (L–M)

Quantitative Predictions & Comparisons I

Unlike qualitative predictions and comparisons, **quantitative predictions and comparisons** are based on numerical data, including probabilities, percents, and part-to-whole or part-to-part comparisons. To make quantitative predictions and comparisons, we must rely on our computation skills. Let's use a previous example, shown below, to learn more.

> A student uses a 12-sided die in a game. The sides of the die are numbered 1–12. If the student rolls the die a total of 100 times, how many times should s/he expect to roll an even number?

To answer the question above, we begin by finding the theoretical probability of rolling an even number. The die has 12 sides, so there are 12 possible outcomes. In addition, six even numbers fall between 1 and 12: 2, 4, 6, 8, 10, 12. With this information, we can set up a ratio to show the theoretical probability of rolling an even number.

$$\frac{\text{number of favorable outcomes}}{\text{number of possible outcomes}} = \frac{6}{12}$$

The theoretical probability of rolling an even number one time is $\frac{6}{12}$, or $\frac{1}{2}$. So what is the theoretical probability of rolling an even number if the student rolls the die 100 times? We compute that answer, as shown below.

$$\frac{1}{2} \times 100 = \frac{100}{2} = 50$$

Our prediction: If the student rolls the die 100 times, s/he should expect to roll an even number 50 times.

Working Together: Work with a classmate to answer the following question. Show your work in the box below.

If the student rolls the die 150 times, how many times should s/he expect to roll the numbers 1, 2, or 3?

Standard 7.1A; 7.1D; 7.1F; 7.1G; 7.7A (L–M)

Non-Proportional Relationships

In previous lessons, you learned that a proportional relationship is a relationship between two quantities in which one quantity is a constant multiple of the other quantity. We use the linear equation shown below to represent a proportional relationship.

$$y = kx$$

Linear equations can also represent non-proportional relationships. A **non-proportional relationship** is a relationship between two quantities, but neither quantity is a constant multiple of the other quantity. The linear equation for non-proportional relationships is shown below. (Note: $b \neq 0$)

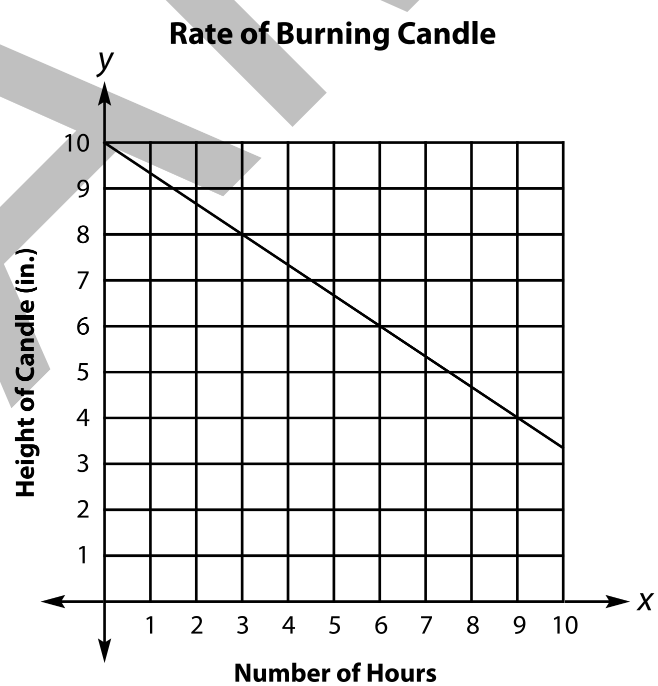
$$y = mx + b$$

In the linear equation above, m represents a rate of change. The rate of change is the ratio of change in the y-values to the change in the corresponding x-values. The formula to find the rate of change appears below.

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

In the linear equation above, b represents an initial value. On a graph, b is the point at which the line crosses the y-axis, also known as the y-intercept.

On a coordinate plane, the graph of a linear equation that represents a non-proportional relationship is always a straight line; however, it does not pass through the origin (0, 0). Look at the example below.



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Standard 7.1D; 7.1F; 7.1G; 7.10B (L–M)

Representing Solutions on a Number Line

You can graph the solution to an equation on a number line. We will begin by finding the value of *y* in the equation below.

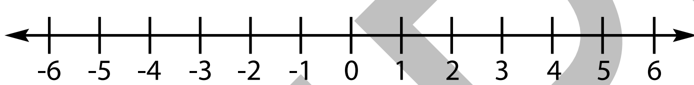
$$5y + 14 = 34$$

On Your Own–1

- Find the value(s) of *y* in the equation. Write your answer below.

$y =$ _______

- Once you know the value(s) of *y*, graph the value(s) on the number line below.



Talk About It–1: How many values did you graph on the number line? Why?

Try It–1: Solve each equation below. On a separate sheet of paper, draw a number line for each equation and graph the equation's solution on the number line.

1. $3x + 5 = 20$ $x =$ ________________
2. $4c - 12 = 16$ $c =$ ________________
3. $0.5t + 0.25 = 2.5$ $t =$ ________________
4. $0.25x + 3.5 = 9.5$ $x =$ ________________
5. $10c + 14 = 54$ $c =$ ________________
6. $5m - 20 = 40$ $m =$ ________________
7. $36p - 8 = 118$ $p =$ ________________
8. $8b - 15 = 145$ $b =$ ________________
9. $40 + 15c = 115$ $c =$ ________________
10. $3n + 22 = 88$ $n =$ ________________

You can also graph the solution to an inequality on a number line. We will begin by finding the value of *x* in the inequality below.

$$4x + 12 > 24$$

continue to next page

Standard 7.1C; 7.1D; 7.1F; 7.1G; 7.11A (L–M)

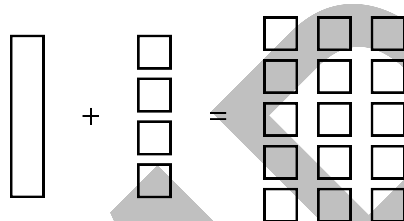
Models for Equations & Inequalities

Models provide another way to understand and solve equations and inequalities. We will use the following simple equation to learn about models.

$$x + 4 = 15$$

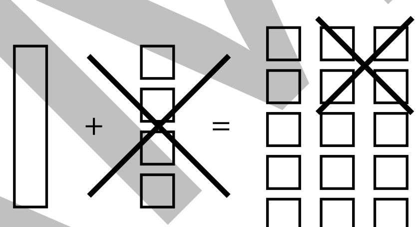
We will use a long bar to represent *x*, the unknown quantity in the equation. We will use small squares to represent the quantities we do know. Our model for this equation will look like the one below.

$x \quad + \quad 4 \quad = \quad 15$



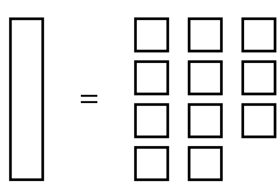
To find the value of *x*, we must "isolate" it on one side of the equation. We can do this by subtracting 4 from both sides. Remember, we must subtract 4 from *both* sides of the equation because each side of the equation must remain equal to the other side.

$x \quad + \quad 4 - 4 \quad = \quad 15 - 4$



We simplify the equation to determine that *x*, our unknown quantity, is 11.

$x \quad = \quad 11$



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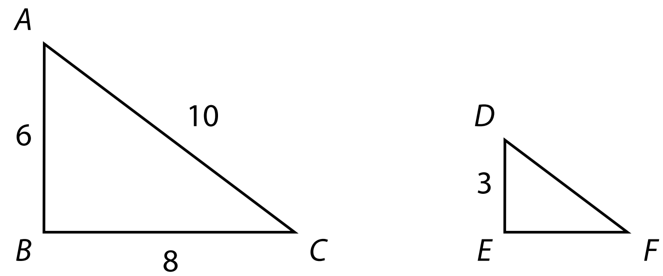
Standard 7.1C; 7.1D; 7.1F; 7.1G; 7.5A (L–M)

A Review of Similarity

In the diagram below, $\triangle ABC$ and $\triangle DEF$ are similar figures.

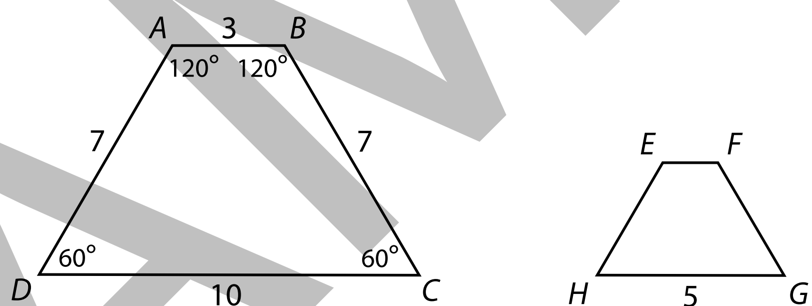


Similar figures have the same shape, but not necessarily the same size. If two figures are similar, their corresponding angles are congruent (equal), and their corresponding sides are all in the same proportion.

Talk About It–1

- Which angles in $\triangle ABC$ and $\triangle DEF$ are congruent?
- How can you determine the length of $\overline{EF}$ and $\overline{DF}$ in $\triangle DEF$?
- What is the length of $\overline{EF}$ and $\overline{DF}$ in $\triangle DEF$?

In the diagram below, Figure *ABCD* and Figure *EFGH* are similar.



Talk About It–2: What is the measure of each angle in Figure *EFGH*?

In the diagram above, we know the length of one pair of corresponding sides: $\overline{CD} = 10$ and $\overline{GH} = 5$. We could write this information as a ratio, as shown below.

$$\frac{CD}{GH} = \frac{10}{5}$$

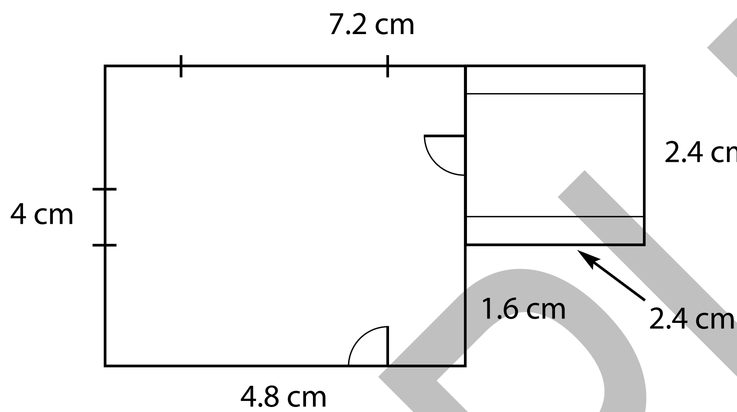
Since $\overline{CD}$ is twice the length of $\overline{GH}$, all the other pairs of corresponding sides are in the same proportion, as well.

Try It: Set up ratios to determine the lengths of $\overline{EF}$, $\overline{FG}$, and $\overline{EH}$ in Figure *EFGH* above. Show your work on a separate sheet of paper.

Standard 7.1A; 7.1D; 7.1F; 7.1G; 7.5C (M)

Working With Scale

Architects use scale drawings called blueprints—the drawings or sketches that represent a finished building's appearance. Blueprints include floor plans that show the specific measurements and layouts for different rooms in a building. The floor plan for a bedroom appears below.



Floor plans usually include a scale used to interpret drawings. Like a scale factor, a **scale** is a ratio between two sets of measurements. However, scales are written differently than scale factors. The floor plan above was drawn according to the following scale.

Scale: 2 cm = 5 ft

The scale shows that 2 centimeters on the floor plan represent 5 feet in the actual room. You can write the scale as a scale factor, as shown below.

$$\frac{2\text{ cm}}{5\text{ ft}}$$

Talk About It–1

- Why is the scale for the floor plan $\frac{2\text{ cm}}{5\text{ ft}}$ and not $\frac{5\text{ ft}}{2\text{ cm}}$?
- How could you find the bedroom's actual dimensions?

To convert the bedroom's actual measurements to the scale drawing's measurements, the architect used a scale of $\frac{2\text{ cm}}{5\text{ ft}}$. To convert the scale drawing's measurements to the room's measurements, you would use the inverse of the scale, or $\frac{5\text{ ft}}{2\text{ cm}}$.

continue to next page

Standard 7.1D; 7.1F; 7.9C (L–M)

Composite Figures & Area

Area is the measure of space inside a two-dimensional figure. Area is measured in square units.

In previous lessons, you have found the area of different two-dimensional figures, including rectangles, triangles, parallelograms, trapezoids, and triangles. Generally, finding the area of such simple figures is a matter of "plugging" a figure's dimensions into a formula and computing an answer. Sometimes, however, we encounter a two-dimensional figure with a difficult area formula, like the one shown below. We can find the area of such figures using a simpler method.

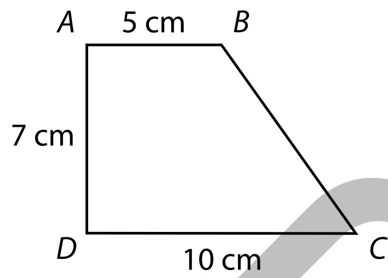
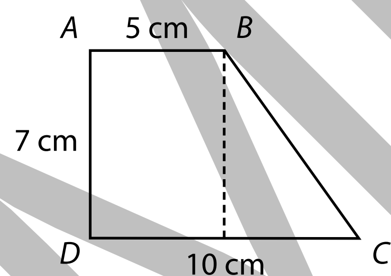


Figure *ABCD* is a composite figure. A **composite figure** is a combination of basic figures. We can draw a line to separate Figure *ABCD* into two basic figures: a rectangle and a triangle.



Now we can find the area of the rectangle and the triangle by using the formula for each shape.

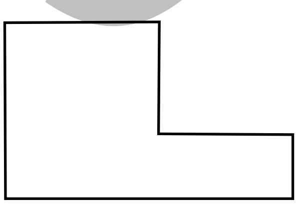
To find the area of the rectangle, we multiply its length times its width: 7 x 5 = 35. The rectangle's area is 35 cm^2.

To find the area of the triangle, we multiply its height times its width, and then divide by 2: 7 x 5 = 35; 35 ÷ 2 = 17.5. The triangle's area is 17.5 cm^2.

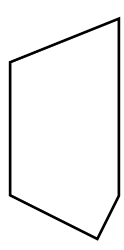
To find the area of Figure *ABCD*, we add the areas of the two basic shapes: 35 + 17.5 = 52.5. The area of Figure *ABCD* is 52.5 cm^2.

Try It: What basic shapes do you see in the composite figures below? Draw dotted lines to divide each composite figure into basic shapes.

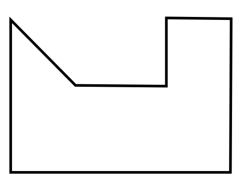
A



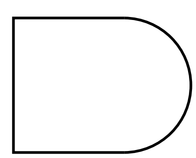
B



C



D



Standard 7.1A; 7.1D; 7.1E; 7.1F; 7.1G; 7.12A (L–M)

Using Box Plots to Compare

You already know that a **box plot** is a statistical diagram that summarizes a data set using the following five plotted points on a number line: minimum, lower quartile (Q1), median, upper quartile (Q3), and maximum.

- The **minimum** is the least value in a data set.
- The **maximum** is the greatest value in a data set.
- The **median** is the middle value in a data set.
- The **lower quartile** is the median of the lower half of a data set.
- The **upper quartile** is the median of the upper half of a data set.

We can also use a box plot to determine the range of values in a data set. Remember, range is the difference between the greatest and least values of a data set.

The box plot below summarizes the data about the football players' weights (page 172).

Try It–1: Use the box plot to identify the minimum, maximum, median, lower quartile, and upper quartile values for the data set. Write your answers below.

Minimum ______ Lower Quartile ______

Maximum ______ Upper Quartile ______

Median ______

Try It–2: Using the basketball players' weights from page 172 as the data set, identify the minimum, maximum, median, lower and upper quartile values, and range. On a separate sheet of paper, create a box plot that displays these values.

a. minimum ____________
b. lower quartile ____________
c. median ____________
d. upper quartile ____________
e. maximum ____________
f. range ____________

Talk About It: Based on the information above, what can you infer about the football players' weights compared to the basketball players' weights?

Standard 7.1A; 7.1D; 7.1E; 7.1F; 7.1G; 7.13B (M)

A Personal Budget

A person's **income** is the money he or she earns for doing a job. An **expense** is an amount of money a person must pay out to others. Expenses may be fixed or variable. A **fixed expense** remains the same from week to week or month to month. A **variable expense** changes from week to week or month to month.

Talk About It–1

- What expenses do most people have?
- What are some examples of fixed expenses?
- What are some examples of variable expenses?
- What are some unexpected expenses that people might have?

A **personal budget** is an individual's plan for managing income and expenses. Ideally, a person wants to have more money coming in (income) than money going out (expenses). At the very least, income and expenses should be equal, or balanced.

Talk About It–2

- Should a personal budget be based on an individual's gross income or net income? Why?
- How might an individual allow for savings, emergencies, and taxes in a personal budget?
- What kinds of savings plans should be included in a personal budget? Why?

Try It

A. Directions: Read about Julie and her plan to create a personal budget. Use the information and what you know about personal budgets to answer the questions that follow.

Julie's Expenses

Expense	Amount Spent
Rent to her parents	$150
Food (dining out)	$230
Cell phone	$105
Transportation (to and from work)	$90
Beauty salon (hair and nails)	$100
Clothes	$225
Entertainment	$90
Daily coffee breaks	$60
Total Expenses	

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