## Standard 6.1D; 6.1F; 6.2A (L-M)

## Classifying Numbers

In the past, you have worked with whole numbers, integers, and rational numbers. A whole number is any of the numbers $0,1,2,3,4,5$, and so on. Whole numbers are never negative and never include a decimal point or a fraction. An integer is any whole number or its opposite ( $\ldots,-3,-2-1,0,1,2,3, \ldots)$. A rational number is any number that can be written as $\frac{a}{b}$ or $-\left(\frac{a}{b}\right)$ for some fraction $\frac{a}{b}$. A rational number can be represented as a point on a number line. Rational numbers include fractions, integers, and decimals that terminate (end) or repeat a pattern of numbers. Examples of rational numbers include $-16,0,245, \frac{3}{5}, 3.58$, and $5 . \overline{5}$.

A Venn diagram can help you classify different types of numbers. A Venn diagram is a drawing that shows relationships among different items. You may have seen Venn diagrams that look like the one to the right. The overlapping part of the two ovals contains the items that are common to both sets.


The Venn diagram for classifying types of numbers looks different. That is because each set of numbers is a part of another set of numbers. The Venn diagram for the number system is shown below.


On Your Own: On the lines below, give five examples of whole numbers, five examples of integers, and five examples of rational numbers.

Whole Numbers: $\qquad$
Integers: $\qquad$
Rational Numbers: $\qquad$

## Absolute Value

Absolute value $(|\mid)$ is the distance of a number from 0 on a number line without considering which side of 0 the number is on. This means the absolute value of a number is never negative. Absolute value is sometimes called magnitude.
Look at the examples below.
What is the absolute value of 3 ?
To answer this question, you must determine how far 3 is from 0 on a number line.


The number 3 is 3 units from 0 on the number line.
The absolute value of 3 is 3 .

$$
|3|=3
$$

What is the absolute value of -2.5 ?
To answer this question, you must determine how far - 2.5 is from 0 on a number line.


The number -2.5 is 2.5 units from 0 on the number line.
The absolute value of -2.5 is 2.5 .

$$
|-2.5|=2.5
$$

Now, look at the two examples below.
|8| The absolute value of 8 is 8 .
$|-8|$
The absolute value of -8 is 8 .
A number and its opposite have the same absolute value because they are both the same distance from 0 on a number line.

## Talk About It

- Explain why -6 is less than -5 but $|-6|$ is greater than $|-5|$.
- Create a rule for finding the absolute value of a positive number.
- Create a rule for finding the absolute value of a negative number.
- What is the absolute value of 0 ? How do you know?
- When the value of a negative number decreases, its absolute value increases. Why?


## Exponents \& Order of Operations

When evaluating an expression or solving an equation, it is important to remember order of operations.

You learned about order of operations in fifth grade, but you did not work with exponents. The list below shows where exponents occur in order of operations.

## Order of Operations

1—any operation inside parentheses
2-any operation inside brackets

## 3-any exponents from left to right

4-multiplication and division worked left to right
5-addition and subtraction worked left to right

What You Need to Know: In fifth grade, you learned that parentheses and brackets are
punctuation symbols used to group things. Parentheses look like this: ( ). Brackets look like this: [].

Look at the example below. It shows the correct order in which to perform the operations.

$$
2+\left(3^{2} \times 4\right)=n
$$

First, compute the exponent inside the parentheses.

$$
\begin{array}{ll}
2+\left(\mathbf{3}^{2} \times 4\right)=n & 3^{2}=9 \\
2+(\mathbf{9} \times \mathbf{4})=n & 9 \times 4=36 \\
\mathbf{2}+\mathbf{3 6}=n & n=38
\end{array}
$$

Second, multiply inside the parentheses.
Finally, add.

$$
2+\left(3^{2} \times 4\right)=38
$$

Try It: Evaluate each expression below. Show all of your work on a separate sheet of paper.

1. $\left(2+3^{2}\right)+\left(2^{4} \times 2\right)$ $\qquad$
2. $\left(3^{3}-10\right) \times(2 \times 7)^{2}$ $\qquad$
3. $10 \times\left[7+\left(85-3^{2} \times 9\right)\right]+2^{3}$ $\qquad$
4. $75-2 \times\left[4 \times\left(12^{2} \div 48+6^{2}\right)\right] \div 8$ $\qquad$

Talk About It: Each time you evaluate an expression, you generate an equivalent expression.
Equivalent expressions have the same value. How many equivalent expressions did you generate for \#1-\#4 above? How do you know?

## Finding Reciprocals

Remember, a reciprocal is one number in a pair of numbers that, when multiplied, have a product of 1 . Multiplicative inverse is another term for reciprocal.

$$
\text { a number } x \text { its reciprocal }=1
$$

An easy way to find the reciprocal of a fraction is to switch the numerator and the denominator. The chart below shows examples of fractions and their reciprocals, as well as a way to check reciprocals.

| Fraction | Reciprocal | Check |
| :---: | :---: | :---: |
| $\frac{1}{4}$ | $\frac{4}{1}$ | $\frac{1}{4} \times \frac{4}{1}=\frac{(1 \times 4)}{(4 \times 1)}=\frac{4}{4}=1$ |
| $\frac{2}{3}$ | $\frac{3}{2}$ | $\frac{2}{3} \times \frac{3}{2}=\frac{(2 \times 3)}{(3 \times 2)}=\frac{6}{6}=1$ |
| $\frac{5}{9}$ | $\frac{9}{5}$ | $\frac{5}{9} \times \frac{9}{5}=\frac{(5 \times 9)}{(9 \times 5)}=\frac{45}{45}=1$ |

Talk About It-1: How can you find the reciprocal of a mixed number (whole number plus a fraction), such as $2 \frac{1}{3}$ ?

On Your Own: Write the reciprocal of each number below.

1. $\frac{3}{4}$
2. $2 \frac{2}{5}$
3. $\frac{7}{5}$
4. $\frac{1}{8}$

5. $1 \frac{5}{6}$
6. $\frac{3}{2}$
7. $3 \frac{3}{5}$
8. $\frac{5}{12}$

Talk About It-2: What is the reciprocal of 3? Explain your answer.

## Multiplying Integers Without Models

There are two important rules for multiplying integers. Look at the table below.
Rules for Multiplying Integers

| Situation | How to solve | Examples |
| :---: | :---: | :---: |
| Two positive numbers; two negative numbers | Multiply the absolute values of the two factors. When the factors have the same sign, the product will be a positive number. | $\begin{array}{ll} 5 \times 2=? & -5 \times-2=? \\ \|5\| \times\|2\|=10 & \|-5\| \times\|-2\|=10 \\ 5 \times 2=10 & -5 \times-2=10 \end{array}$ |
| A positive number and a negative number | Multiply the absolute values of the two factors. When the factors have different signs, the product will be a negative number. | $\begin{array}{ll} 5 \times-2=? & -5 \times 2=? \\ \|5\| \times\|-2\|=10 & \|-5\| \times\|2\|=10 \\ 5 \times-2=-10 & -5 \times 2=-10 \end{array}$ |

You can use these rules to multiply any two integers.

On Your Own: Solve each problem below.

1. $-2 \times 3=$ $\qquad$
2. $7 \times(-1)=$ $\qquad$
3. $4 \times(-2)=$ $\qquad$ 6. $1 \times(-7)=$ $\qquad$
4. $6 \times 3=$ $\qquad$
5. $8 \times 2=$ $\qquad$
6. $-5 \times 3=$ $\qquad$
7. $8 \times(-2)=$ $\qquad$

Working Together: Read the problem below. Work with a partner to write the equation and solve the problem. You will share your work with your classmates.

Each member of a golf team has a score of -8 at the end of the first round of play. If there are 6 members on the team, what is the team's combined score at the end of the first round?

Equation: $\qquad$ Answer: $\qquad$

## How to Find Scale Factors

In the previous activity, you created a scale drawing when given a figure and a scale factor. Now you will determine the scale factor when given a figure and a scale drawing.

The diagram below shows Figure A and its scale drawing.

10.5 in.

6 in.

The rectangles above are similar and, therefore, proportional. To find the scale factor used to create the scale drawing, you will use the dimensions of both rectangles. You may choose between two methods to accomplish this.

Remember: The scale factor is the proportional ratio between the lengths of the corresponding sides of a figure and a scale drawing of the figure.

## Method \#1

Set up a ratio to compare the widths of both rectangles to determine the scale factor used to create the scale drawing. The scale drawing's width will be the top number of the ratio, and the original figure's width will be the bottom number of the ratio.

$$
\frac{\text { scale drawing's width }}{\text { original figure's width }}=\frac{6 \mathrm{in} .}{2 \mathrm{in} .}=\frac{3}{1}=3
$$

The scale factor used to create the scale drawing is 3.

## Method \#2

Set up an equation to determine the scale factor used to create the scale drawing. To do this, you use the corresponding lengths of Figure $A$ and its scale drawing.
You know that the length of Figure A multiplied by the scale factor equals the length of the scale drawing. You can write this as an equation, as shown below.

Figure A's length $\times$ scale factor $=$ scale drawing's length

Standard 6.1A; 6.1F; 6.4H (L-M)

## Conversion Practice

Directions: Solve each problem below. Show all of your work.

1. How many inches are in 3 miles, given that 1 yard $=36$ inches?

Answer: $\qquad$
2. How many meters are in 19 miles, given that 1 foot $\approx 0.305$ meters?

Answer: $\qquad$
3. How many liters are in 7 pints, given that 1 cup $\approx 0.237$ liters?

Answer:

4. How many grams are in 10 pounds, given that 1 ounce $\approx 28.35$ grams?

Answer: $\qquad$
5. Challenge! How many kilometers are in 2,145 feet, given that 1 mile $\approx 1.6$ kilometers?

Answer: $\qquad$
6. Challenge! How many tablespoons are in 21 cups, given that 1 quart $=64$ tablespoons?

Answer: $\qquad$

## Area Formulas

## Area of a Triangle

You find the area of a triangle by multiplying $\frac{1}{2}$ the length of its base (b) times its height $(h)$. Look at the triangle to the right.


Let's find the area.

$$
\begin{gathered}
A=\frac{1}{2} b h \\
A=\frac{1}{2}(14 \times 6)
\end{gathered}
$$

$$
A=42 \mathrm{~cm}^{2}
$$

Try It-1: Find the area of the triangle below.


Answer: $\qquad$ square units

## Area of a Trapezoid

You already know how to find the area of rectangles and triangles. You can use this information to find the area of a trapezoid. A trapezoid is a four-sided figure with two parallel sides (sides that stay the same distance apart and never meet). Look at the trapezoid below.

The dashed line in the trapezoid divides the shape into a rectangle and a triangle. The area of the trapezoid is the sum of the areas of the rectangle and triangle.


$$
\begin{gathered}
A=l w+\frac{1}{2} b h \\
A=(4 \times 7)+\frac{1}{2}(5 \times 7)
\end{gathered}
$$

$$
A=28+17.5
$$

$A=45.5$ square units

Standard 6.1A; 6.1E; 6.1F; 6.8D (M-H)

## Finding Volume

A. Directions: Read the information below, and answer the questions that follow.

The boxes in a warehouse are arranged by size. Stacks of products form rows in the warehouse. Each row is assigned a letter of the alphabet.
Each row in the warehouse is 45 feet long. Each stack of products in a row is 8 feet wide and 20 feet tall. The stacks of products use all of the space along the length of each row.

1. Draw a picture of one row of the warehouse. Label the length, width, and height of the row.
2. Row A contains only boxes that measure 1 foot $x 1$ foot $x 1$ foot. When this row is completely filled, how many boxes are in it? $\qquad$
3. Row $B$ contains boxes that measure $\frac{1}{2}$ foot $x \frac{1}{2}$ foot $x \frac{1}{2}$ foot. When this row is completely filled, how many boxes are in it? $\qquad$
4. If there are 14,400 boxes in row $C$, what is the volume of each box? $\qquad$

## Outliers

A math teacher listed her students' scores on a math quiz, in order from lowest to highest.

$$
\begin{array}{llllllllllllllll}
74 & 76 & 76 & 76 & 76 & 78 & 78 & 80 & 80 & 80 & 80 & 80 & 80 & 80 & 81 & 81 \\
100
\end{array}
$$

Working Together: Work with a classmate to find the mean and median score for the data set above. (Round the mean to the nearest whole number.) Write your answers in the spaces below.

Mean: $\qquad$ Median: $\qquad$

## Talk About lt-3

- How well does the mean represent the students' quiz scores overall?
- How well does the median represent the students' quiz scores overall?
- Do any values "stand out" as unusual in the data set? Why?

In the data set above, the score of 100 is an outlier. An outlier is a value that "lies outside" most of the other values in the data set. In other words, the value is either much less or much more than the other values.

On Your Own-2: The data set below shows the number of points some children earned in a summer reading program. Find the mean and median number of points for the data set. Then, answer the questions that follow.

| 2 | 2 | 2 | 3 | 3 | 3 | 4 | 6 | 6 | 14 | 14 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |

a. What is the median for this data set? $\qquad$
b. What is the mean for this data set? (Round to the nearest whole number.) $\qquad$
c. What are the outliers in this data set? $\qquad$
d. Do the outliers affect the mean for this data set? If so, what effect do they have? $\qquad$
e. Do the outliers affect the median for this data set? If so, what effect do they have? $\qquad$
$\qquad$
f. Which measure of center, mean or median, more accurately represents the center of this data?

Why? $\qquad$

Talk About lt: Describe some situations in which a person might write a check. Why might a person want to pay by check rather than with cash?

Banks charge money for the services that they provide. Let's talk about some common fees.
An account fee (or service fee) is usually a monthly fee charged to keep an account open. Some banks waive the fee if a customer maintains a minimum balance, sets up direct deposit, or receives online-only bank statements. Sometimes banks offer free checking accounts to compete for your business!

Debit and ATM fees are charged when you complete a transaction using a debit card at a cash register or at an ATM. Some banks charge for debit card transactions, while others do not. A bank generally charges an ATM fee when a customer uses another bank's ATM, but the fee can be waived or reduced if the customer uses that bank's ATM.

An overdraft fee is charged for making a withdrawal that is greater than the account balance (insufficient funds). If you write a check for more money than you have in your account, you must come up with the money for the goods or services AND a fee of $\$ 10$ to $\$ 25$ to the merchant for the trouble AND an additional $\$ 10, \$ 20$, or even $\$ 30$ in overdraft fees to the bank! It's best not to overdraft at all!

Many banks offer overdraft protection, meaning the bank will pay for a debit purchase even if you do not have enough money in the account. There is usually a fee from $\$ 10$ to $\$ 35$ for each purchase that is an overdraft. Many banks will also transfer money from your savings or other account to cover an overdraft from a checking account.

On Your Own: The table below shows the fees for certain features at three banks.

|  | Filmore National Bank | Twin Pines City Bank | First National Bank |
| :---: | :---: | :---: | :---: |
| Online <br> Banking | Free | Free | Free |
| Checking | \$10 per month <br> checking fee | \$0.50 per check | Free checking |
| Debit Card | \$0.75 per debit card <br> transaction | No debit card fees | \$1 per debit card <br> transaction |
| ATM <br> Transactions | No ATM fees | Nonk ATM: \$1 <br> Bank ATM: \$2 per <br> transaction | \$2.75 per transaction |

1. Sally is looking for a new bank. In an average month, Sally writes 7 checks, makes 2 bank and 2 non-bank ATM transactions, and 4 debit card purchases.

Which bank should she choose? $\qquad$ Why?

